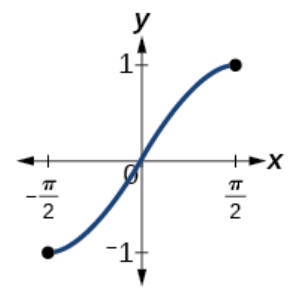
# Understanding and Using the Inverse Sine, Cosine, and Tangent Functions

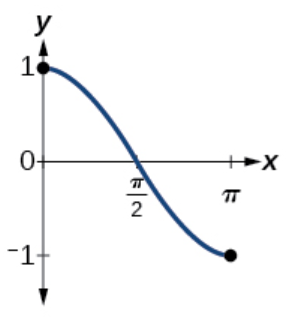
An inverse trig function “undoes” what the original trigonometric function “does,” as is the case with any other function and its inverse.

Recall that the domain of the inverse function is the range of the original function, and vice versa. In other words, for a one-to-one function, if , then an inverse function would satisfy .

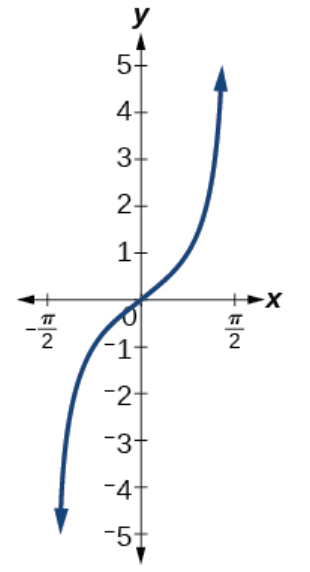
However, sine, cosine, and tangent functions are not one-to-one functions. The graph of each of these functions would fail the horizontal line test. No periodic function can be one-to-one because each output in its range corresponds to at least one input in every period, and there are an infinite number of periods. Since they are not one-to-one, we need to restrict the domain of each function to yield a “restricted” function that is one-to-one.



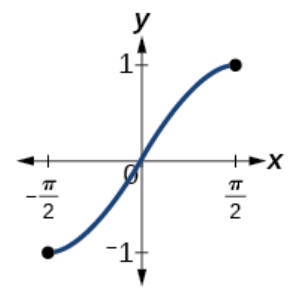
Sine function restricted to the domain of



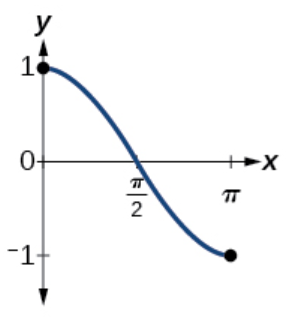
Cosine function restricted to the domain of



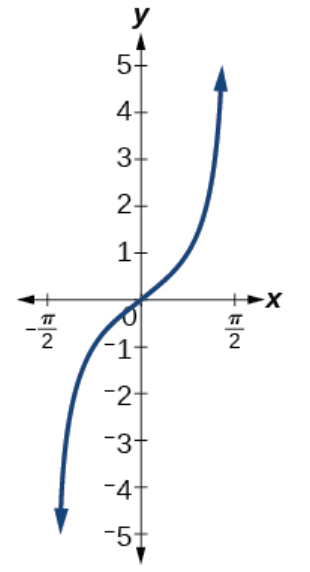
Tangent function restricted to the domain of



Sine function restricted to the domain of



Cosine function restricted to the domain of



Tangent function restricted to the domain of

On these restricted domains, we can define the inverse trigonometric functions:

• The **inverse sine function,** , means . The inverse sine function is also called the **arcsine** function and is notated .

has domain and range

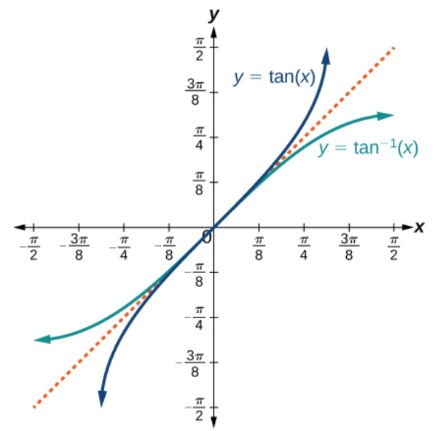
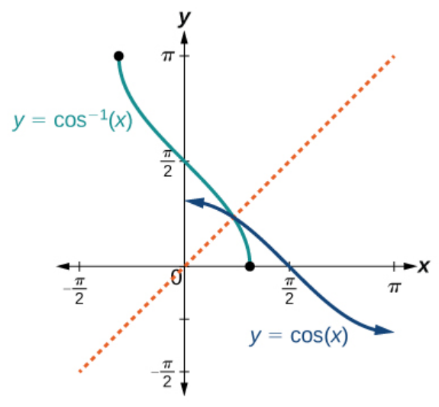
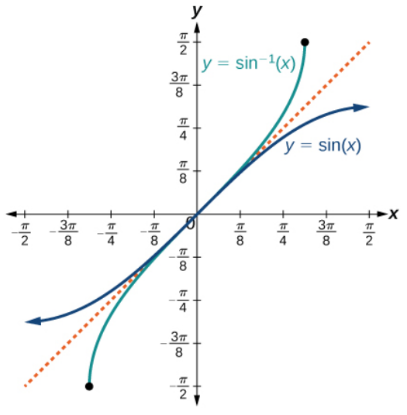
• The **inverse cosine function,** , means . The inverse cosine function is sometimes called the **arccosine** function and is notated .

has domain and range

• The **inverse tangent function,** , means . The inverse tangent function is sometimes called the **arctangent** function and is notated .

has domain and range

In addition, just like with other inverse functions, the graph of an inverse trigonometric function is a reflection of the graph of the original function about the line .



Examples

1. Given , write a relation involving the inverse sine.
2. Given , write a relation involving the inverse cosine.

# Finding the Exact Value of Expressions Involving the Inverse Sine, Cosine, and Tangent Functions

Now that we can identify inverse functions, we can evaluate them. Just like we did with the original trigonometric functions, we can give exact values for the inverse functions when we are using the “special” angles, specifically , , and , and their reflections into other quadrants.

Given a “special” input value, evaluate an inverse trigonometric function.

1) Find angle for which the original trigonometric function has an output equal to the given input for the inverse trigonometric function.

2) If is not in the defined range of the inverse, find another angle that is in the defined range and has the same sine, cosine, or tangent as , depending on which corresponds to the given inverse function.

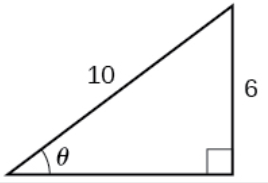
Examples: Evaluate each of the following.

# Using a Calculator to Evaluate Inverse Trigonometric Functions

To evaluate inverse trigonometric functions that do not involve the “special” angles, we use a calculator or other type of technology. Most calculators have specific keys or buttons for the inverse sine, cosine, and tangent functions.

Examples

1. Evaluate .
2. Evaluate .
3. Use the diagram below to find the angle .



1. Suppose a 13-foot ladder is leaning against a building, reaching to the bottom of the second-floor window 12 feet above the ground. What angle, in radians, does the ladder make with the building?
2. The line passes through the origin in the plane. What is the measure of the angle that the line makes with the negative axis?

# Finding Exact Values of Composite Functions with Inverse Trigonometric Functions

There are times when we need to compose a trigonometric function with an inverse trigonometric function. In these cases, we can usually find exact values for the resulting expressions without resorting to a calculator.

## Evaluating Compositions for the Form and

Compositions of a Trigonometric Function and Its Inverse

for

for

for

only for

only for

only for

\*\*Notice the restriction in the domain! Refer back to the inverse graphs.

Examples: Evaluate each of the following.

## Evaluating Compositions of the Form

Now that we can compose a trigonometric function with its inverse, we can explore how to evaluate a composition of a trigonometric function and the inverse of another trigonometric function.

Given functions of the form and , we can evaluate them by

1) If is in , then .

2) If is not in , then find another angle in such that .

3) If is in , then .

4) If is not in , then find another angle in such that .

Examples

1. Evaluate .
2. Evaluate .

## Evaluating Compositions of the Form

For these types of compositions, we may need to use the trigonometric relations between the angles and the sides of a right triangle, together with the use of Pythagoras’s relation between the lengths of the sides. We can also use the Pythagorean identity, , to solve for one when given the other, as well as inverse trigonometric functions to find compositions involving algebraic expressions.

Examples

1. Find an exact value for .
2. Evaluate .
3. Find an exact value for .
4. Find a simplified expression for for .